MATH 201: Linear Algebra – Week 14 Quiz

NAME: Solutions ID Number: _____

Problem 1 (3 points)**.** Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}.$$

(a) Write the characteristic equation for A.

(b) Find *all* the eigenvalues of A and determine their algebraic multiplicities.

$$f_A(\lambda) = (2-\lambda)[(1-\lambda)^2 - 4] = (2-\lambda)(\lambda-3)(\lambda+1) = 0$$

$$\lambda_1 = 2, \qquad \lambda_2 = 3, \qquad \lambda_3 = -1$$

Each eigenvalue has algebraic multiplicity 1

Problem 2 (3 points)**.** Suppose that a 5×5 matrix A has the following eigenvalues

- $\lambda_1 = 1$ with *algebraic multiplicity* 2.
- $\lambda_2 = 2$ with algebraic multiplicity 1
- $\lambda_3 = 4$ with algebraic multiplicity 2.

Compute $\det A$ and $\operatorname{tr} A$.

$$\det A = 1^2 * 2 * 4^2 = 2 * 16 = 32$$

$$\operatorname{tr} A = 1 + 1 + 2 + 4 + 4 = 12$$

Problem 3 (4 points)**.** A mathematical model tracks two deer populations – juveniles and adults – with state vector $\vec{x}(t) \in \mathbb{R}^2$. The model satisfies $\vec{x}(t+1) = A\vec{x}(t)$ where A is a 2 by 2 matrix. Suppose that the initial condition, $\vec{x}(0) = \vec{v}$, is an eigenvector of A with eigenvalue $\lambda = 1.2$.

(a) Does the overall population grow, shrink, or stay the same over time?

(b) What value of λ would correspond to a steady population

Briefly justify your answers.

(a) Note that $x(t) = A^t x(0) = A^t v = (1.2)^t v$.

Thus each component of the vector v grows and hence the total population, which is their sum, also grows.

(b) a steady population would mean that $x(t) = x(t+1)$

This will happen when $\lambda = 1$ by the same logic as above.