MATH 201: Linear Algebra – Week 13 Quiz

NAME: Solutions

ID Number: ____________

Problem 1 (3 points). Suppose that A is a 4 by 4 matrix with $\det A = 3$. Let $\vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\vec{v}_4$ be the column vectors of A. Suppose that B is obtained from A by swapping two rows and then multiplying one row by -2. What is $\det(B)$?

row swap $\longrightarrow (-1)$

multiply by $-2 \longrightarrow (-2)$

$$\boxed{\Rightarrow \det(B) = (-1)(-2)\det(A) = 2 \cdot 3 = 6}$$

Problem 2 (3 points). Let A, B be $n \times n$ matrices with $\det(A) = x$. Suppose

$$\det\left(AB^{\top}A^{-1}\right) = y.$$

What is $\det(B)$ in terms of x and y? Be sure to show your work.

$$\det(AB^{\top}A^{-1}) = \det(A)\det(B^{\top})\det(A^{-1})$$

$$= \det(A)\det(B)\frac{1}{\det(A)}$$

$$= \frac{\det(A)}{\det(A)}\det(B)$$

$$= \det(B)$$

$\Rightarrow$ as long as $x \neq 0$, $\det(B) = y$.

If $x = 0$, then there is not enough information.

Problem 3 (4 points). Suppose that the vectors $\vec{u}, \vec{v} \in \mathbb{R}^2$ span a parallelogram with area 4. What is the area of the parallelogram spanned by the vectors

$$2\vec{u} \quad \text{and} \quad \vec{u} + \vec{v}\ ?$$

Since the area spanned by $\vec{u}$ and $\vec{v}$ is nonzero, they are linearly independent. thus, we may consider $\mathcal{B} = \{\vec{u}, \vec{v}\}$ as a basis for $\mathbb{R}^2$. then $[2\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $[\vec{u}+\vec{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, while, of course, $[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Thus the matrix sending

$$A: \vec{u} \longmapsto 2\vec{u}$$
$$\vec{v} \longmapsto \vec{u} + \vec{v}$$

is $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$. We have $\det(A) = 2 \Rightarrow \boxed{\text{new area} = 2 \cdot 4 = 8}$